

## **CAMERA CALIBRATION SYSTEM USING PLANAR CONCENTRIC CIRCLES AND METHOD THEREOF**

### TECHNICAL FIELD

5        The invention relates generally to a method of calibrating a camera for obtaining the camera parameters required when the camera is used to obtain geometric information. More particularly, the invention relates to a camera calibration system and method thereof, which use a concentric circle pattern as an artificial calibration pattern in the process of calibrating the camera and obtain camera parameters by  
10      analyzing two images of the concentric circle pattern taken at different angles.

### BACKGROUND OF THE INVENTION

To obtain geometric information of an object using a camera, it is required to calibrate the camera by estimating parameters between the image information obtained from the camera and the actual geometric information of the object. Fig. 1  
15      illustrates an image of a 3D artificial calibration object model in a conventional method of calibrating a camera using 3D calibration object and Fig. 2 is an image of a plane model in a conventional method of calibrating a camera using coordinates of points on a two-dimensional plane.

Conventional methods for calibrating a camera includes ① a calibration  
20      method using a 3D calibration object, ② self calibration method, ③ a calibration method using coordinates of points on a two-dimensional plane.

First, a calibration method widely used so far is one to calibrate the camera using a 3D artificial calibration object, such as a rectangular parallelepiped calibration object, as shown in Fig. 1. As can been seen from Fig.1, a photograph of

the rectangular calibration object is taken to obtain a geometric relation of the rectangular parallelepiped object.

However, in the method of calibrating a camera using a 3D artificial calibration object, it is difficult to manufacture and maintain the calibration object  
5 of a rectangular parallelepiped. This is because the calibration object should have the characteristic of a normal rectangular parallelepiped in order to calculate the parameters of the camera from its image. In other words, it must be a rectangular parallelepiped in which three plane and twelve edges must maintain a right angle with respect to a vertex. Otherwise, exact calculation of the camera parameter  
10 becomes difficult and the reliability of calibration is degraded accordingly.

Second, the self calibration method is one to calculate the parameters of the camera using only information of each of corresponding points from several sheets of images. Although this self calibration method can be widely applied without the limitation of using an artificial calibration object, it is difficult to exactly define the  
15 corresponding points. Due to this, this method has problems that the process of finding a solution is very complicated and it is also difficult to find a correct solution.

Third, as shown in Fig.2, the method of calibrating the coordinates of the points on a plane is relates to the calibration method using artificial calibration  
20 object and the self calibration method. As shown in Fig. 2, this method estimates the parameters of the camera by taking several images of the plane pattern to exactly find the coordinates of the points on a plane and the coordinates of the corresponding points on a corresponding image plane.

For this method of using the coordinates of the points on a plane, one can  
25 manufacture and maintain a 2D plane pattern easier than the method of using a 3D artificial calibration object. However, this method is very complicated since it must use a plurality of calibration points of the images of a plane and compare them to each other.

## SUMMARY OF THE INVENTION

It is therefore an object of the present invention to provide a system for calibrating internal parameters using projection invariable characteristics of a concentric circle and a method thereof, which is able to use a plane pattern and  
5 increase the reliability of calibration, and calculate parameters of a camera without identifying the corresponding points.

In order to accomplish the above object, a method of calibrating a camera by calculating camera parameters required to obtain geometric information of an object using projective invariant characteristic of a concentric circle is characterized in that  
10 it comprises the steps of; a) taking a plurality of images of a calibration pattern consisting of two or more concentric circles located at a same plane and having different radii at different angles to obtain images of ellipses as a result of the projection of said concentric circles; b) calculating the central points of the projected concentric circles using a given algorithm; and c) calculating the principal  
15 point and focal length of said camera for tracing the location of a circle in a 3D space using a nonlinear minimizing method based on the central points calculated in said step b).

Also, a system for calibrating internal parameters of a camera for calibrating parameters between an actual object and images of the object is characterized in that  
20 it comprises a camera; a calibration pattern located on a same plane and consisting of two or more concentric circles having different radii, the image of which is taken by said camera; and a controller for calculating a straight line connecting the central points of the images of ellipses obtained by projecting said calibration pattern of concentric circles to said camera at a given angle, and finding crossing  
25 points of said straight line and said projected ellipses to obtain the central point coordinate of the concentric circle using a cross ratio (Cr), and tracing the location of the circle in a 3D space based on the coordinate of the central point of the concentric circle to calculate a principal point and focal length of the camera.

## BRIEF DESCRIPTION OF THE DRAWINGS

The aforementioned aspects and other features of the present invention will be explained in the following description, taken in conjunction with the accompanying drawings, wherein:

5 Fig. 1 illustrates an image of a 3D artificial calibration object model taken in a conventional method of calibrating a camera using a 3D calibration object;

Fig. 2 is an image of a plane model taken in a conventional method of calibrating a camera using coordinates of points on a two-dimensional plane;

10 Fig. 3 is a conceptual view showing that images of a concentric circle pattern is taken at different angles in a method of calibrating camera parameters using a concentric circle pattern according to one embodiment of the present invention;

Fig. 4a illustrates two images showing a concentric circle taken at different angles in Fig.3, and Fig. 4b illustrates images where boundary lines are extracted from the ellipses in Fig.4a;

15 Fig. 5 is an image showing the central points of the ellipses in Fig.4b;

Fig. 6 is an image showing a straight line connecting the central points of the ellipses in Fig.5; and

Fig. 7 illustrates a position relation of the points on the straight line in Fig.6.

## DETAILED DESCRIPTION OF THE INVENTION

20 The present invention will be described in detail by way of a preferred embodiment with reference to accompanying drawings, in which like reference numerals are used to identify the same or similar parts.

25 Fig. 3 is a conceptual view showing that images of a concentric circle pattern is taken at different angles in a method of calibrating camera parameters using a concentric circle pattern according to one embodiment of the present invention; Fig. 4a illustrates two images showing a concentric circle taken at different angles in Fig.3, and Fig. 4b illustrates images where boundary lines are extracted from the

ellipses in Fig.4a; Fig. 5 is an image showing the central points of the ellipses in Fig.4b; Fig. 6 is an image showing a straight line connecting the central points of the ellipses in Fig.5; and Fig. 7 illustrates a position relation of the points on the straight line in Fig.6.

5 As shown in Fig.3, a concentric circle located in a 3D space has the following geometric conditions.

1) Two circles are located at the same coordinate, 2) two circles are located at the same plane, 3) the radii of two circles are different. An image of the concentric circle having this geometric structure is taken using a camera.

10 Meanwhile, a concentric circle consisting of a circle having a smaller radius and a circle having a larger radius including the smaller circle has the same normal vector and a vertical distance according to above condition 2).

Two images of this concentric circle pattern are taken by one camera at different angles to obtain two sheets of image information. A concentric circle

15 projected at a given angle has a shape of an ellipse, as shown in Fig.4a to Fig.5. Generally, the central points of two ellipses do not match. This is because two ellipses obtained by projecting concentric circles at different angles have different shapes according to the angle of projection. The slant of the straight line including the central points of the two ellipses varies depending on the projection angle of the  
20 camera. Meanwhile, in the drawings, the central point of an ellipse is a middle point of the straight line connecting two focuses, that is, the point at which the long axis and short axis of the ellipse meet.

Meanwhile, as mentioned above, the straight line connecting the central points of the ellipses projected from a 3D concentric circle can be represented by the  
25 following Equation 1.

First, a projection transformation ( $T$ ) into a homogenous coordinate system widely used in image processing and projection geometry can be represented into a projection matrix as Equation 1.

**【 Equation 1 】**

$$T = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix}$$

According to invariable characteristic of a circle which is a trace of points located at a constant distance (radius) from a central point, two concentric circles satisfying the above geometric condition can be represented by following Equation 2.

**【 Equation 2 】**

$$(X - x_0)^2 + (Y - y_0)^2 = r^2, \quad Z = 0$$

10

where,  $x_0$  and  $y_0$  are the central point coordinates of the circle,  $r$  is the radius of the circle and  $Z$  is a value of the vertical axis with respect to the plane.

At this time, using the condition  $Z=0$ , a general projection equation is converted into a two-dimentional transformation (II) between an image plane and a plane where  $Z=0$  as in Equation 3.

**【 Equation 3 】**

$$\begin{bmatrix} sx \\ sy \\ s \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ p_{31} & p_{32} & p_{33} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{14} \\ p_{21} & p_{22} & p_{24} \\ p_{31} & p_{32} & p_{34} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

where,  $X, Y$  indicate actual 3D coordinates in projection geometry theory,  $x, y$  indicate coordinates in the projected image,  $s$  is a scale factor, and  $p$  is a value of the projection transformation matrix.

Also, rearranging Equation 3 in order to obtain an equation of the projected circle Equation 4 is obtained.

**【 Equation 4 】**

$$s \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & p_{14} \\ p_{21} & p_{22} & p_{24} \\ p_{31} & p_{32} & p_{34} \end{bmatrix}^{-1} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

5

Considering the homogeneous characteristic of the coordinate system in Equation 4, multiplying terms of Equation 4 are deleted and Equation 2 is substituted to obtain the following Equation 5.

**【 Equation 5 】**

$$10 \quad \left( \frac{\alpha_{11}x + \alpha_{12}y + \alpha_{13}}{\alpha_{31}x + \alpha_{32}y + \alpha_{33}} - x_0 \right)^2 + \left( \frac{\alpha_{21}x + \alpha_{22}y + \alpha_{23}}{\alpha_{31}x + \alpha_{32}y + \alpha_{33}} - y_0 \right)^2 = r^2$$

Meanwhile, Equation 5 may be rearranged to obtain an equation of an ellipse. Then, the coordinate of the central point of the projected ellipse can be obtained by following Equation 6.

15    **【 Equation 6 】**

$$(X', Y') = \left( \frac{C' + r^2 D'}{A' + r^2 B'}, \frac{E' + r^2 F'}{A' + r^2 B'} \right)$$

Where, A', B', C', D', E' and F' consist of a combinations of factors  $\alpha_{ij}$  in Equation 5 and r is a radius constant of the circle.

20    As can be seen from Equation 6, A', B', C', D', E' and F' are independent from r.

A straight line connecting the central points of the ellipses is defined as Equation 7 using the coordinates of the central points of the ellipses calculated by Equation 6.

【 Equation 7 】

5. 
$$(B'E' - A'F')X' + (A'D' - B'C')Y' + (D'E' + F'C') = 0$$

Equation 7 is an equation of the straight line including only constant coefficients with respect to r. This means that the central points of the ellipses obtained as a result of the projection is located on the straight line defined

10 independently from the radius of the circle, where X', Y' are coordinates of the centers of the ellipses. As the center of the projected concentric circle is same to the center of the ellipses when r=0 in Equation 2, the center of the circle is also located on the straight line defined by Equation 7.

From the relation that the central points of the projected ellipses are located  
15 on one straight line, an algorithm to find the coordinate of the central point of the circle will be now described in detail.

It is defined that the crossing points of the projected internal ellipse and the straight line are A, B, the crossing points of the external ellipse the straight line are A', B', the central point of the circle is O and a given point separated infinitely on  
20 the straight line is M<sub>∞</sub>. Location of these points can be found in Fig.6 and Fig.7 and this positional relation is same in 3D and 2D spaces.

Meanwhile, a given point infinitely separated from the center of the circle O on the straight line is a point virtually set and is frequently used in projection theory.  
This point is typically used because it is easy to serve as the boundary value of a  
25 condition and it has a characteristic limiting the cross ratio.

Therefore, a cross ratio Cr (A, O, B and M<sub>∞</sub>) and Cr (A', O, B' and M<sub>∞</sub>) are same, which is calculated in the same manner in a 3D space by Equation 8.

【 Equation 8 】

$$Cr(A, O, B, M_\infty) = \frac{\overline{BAM_\infty O}}{\overline{BOM_\infty A}} = 2$$

$$Cr(A', O, B', M_\infty) = \frac{\overline{B'A'M_\infty O}}{\overline{B'OM_\infty A'}} = 2$$

Where, the points that can be found in the image are A, B, A' and B', and

5 Equation 8 has two variables of points O and  $M_\infty$ . These two points are commonly used in the two Equations of  $Cr(A, O, B$  and  $M_\infty)$  and  $Cr(A', O, B'$  and  $M_\infty)$ . Therefore, two variables and two equations can be obtained. As a result, the coordinate of point O can be obtained by solving the two Equations.

10 All the items of below Equation 9 can be found from thus obtained coordinate of the central point of the circle. Also, calibration is performed by finding internal parameters  $u_0$ ,  $v_0$  and  $f$  which minimize the value of Equation 9. Calibration of the internal parameters uses below Equation 9.

【 Equation 9 】

$$F(u_0, v_0, f) =$$

$$\alpha(\overline{n_1} - \overline{n_2})^2 + \beta(\overline{n_3} - \overline{n_4})^2 + \gamma(d_1 - d_2)^2 + \lambda(d_3 - d_4)^2$$

$$+ \rho_1 \sum |R_{C_1}(n_1, d_1) - R_1| + \rho_2 \sum |R_{C_2}(n_2, d_2) - R_2|$$

$$+ \rho_3 \sum |R_{C_3}(n_3, d_3) - R_3| + \rho_4 \sum |R_{C_4}(n_4, d_4) - R_4|$$

15

In Equation 9,  $u_0$ ,  $v_0$  and  $f$ , respectively indicate a principal point and a focal length of the camera. The first two items ( $\alpha(\overline{n_1} - \overline{n_2}) + \beta(\overline{n_3} - \overline{n_4})$ ) indicate that two concentric circles have the same normal vector and next two items ( $\gamma(d_1 - d_2) + \lambda(d_3 - d_4)$ ) indicate that two concentric circles are located on the plane

having the same distance. In addition, next four items  
 $(\rho_1 \sum |R_C(n_1, d_1) - R_1| + \rho_2 \sum |R_C(n_2, d_2) - R_2| + \rho_3 \sum |R_C(n_3, d_3) - R_3| + \rho_4 \sum |R_C(n_4, d_4) - R_4|)$   
indicate that a radius obtained from given internal parameters is same as the actual radius.

5

### Experimental Example

A concentric circle is drawn on the plane so that the central points of two circles having a radius of 40mm and 80 mm, respectively are located at one point. Also, as shown in Fig.4a and Fig.4b, images of the circle is taken at two different angles to obtain two images. At this time, a XC-003 camera manufactured by SONY and with focal radius of 8mm was used.

After obtaining two projection images (containing the boundary lines), a calibration value as shown in Table 1 is obtained by means of a camera calibration method using the concentric circle pattern according to the present invention.

15 【Table 1】

Class	$u_O$	$v_O$	$\alpha_u$	$\alpha_v$
Comparative example (Z. Zhang)	306.70	240.44	1104.2	1112.5
Embodiment of the present invention	309.37	246.01	1104.7	1110.7

The comparative example (Z.Zhang) in Table 1 shows the calibrated values according to the camera calibration method taught in an article Z. Zhang, "Flexible  
20 Camera Calibration by Viewing a Plane From Unknown Orientations", *In Proc. 7th International Conference Computer Vision, Corfu, Greece, pp.666-673, September*

1999. Table 1 shows that the calibration value calculated using a concentric circle pattern is similar to the calibration value in the comparative example of Z. Zhang

As mentioned above, the method of calibrating internal parameters of a camera using a concentric circle pattern according to the present invention provides  
5 the calibration parameters of the camera from images of a 2D concentric circle taken from different angles. Thus, the present invention has an advantage that a concentric circle pattern which is easy to manufacture and maintain.

Also, the method of calibrating internal parameters of a camera using a concentric circle pattern according to the present invention calculates camera  
10 calibration parameters using a plurality of Equations. Therefore, the present invention has an advantage that it can easily calibrate a camera by calculating the calibration parameters.

The present invention has been described with reference to a particular embodiment in connection with a particular application. Those having ordinary  
15 skill in the art and access to the teachings of the present invention will recognize additional modifications and applications within the scope thereof. It is therefore intended by the appended claims to cover any and all such applications, modifications, and embodiments within the scope of the present invention.